C. U. Shah Science College, Ashram Road, Ahmedabad

Internal Test

B.Sc. Semester 6 **Physics** **Paper-307**

Date: 16-03-2015 **Time: 1 hr 45 min**

Q-1 (a) Prove that the solutions of Bessel's differential equation $J_m(x)$ and $J_{-m}(x)$ are linearly independent if 'm' is not an integer. (6)

OR

(a) Prove the Rodrigue's formula $P_l(x) = \frac{1}{2^l\, l!} \frac{d^l}{dx^l}(x^2-1)^l$, where l = positive integer (6)

(b) Prove that the solutions of Bessel's differential equation $J_m(x)$ and $J_{-m}(x)$ are linearly dependent if 'm' is an integer. (7)

OR

(b) Show that (i) $\int_{-1}^{+1} x^m P_n(x)dx = 0$, for $m < n$, (7)

and (ii) $\int_{-1}^{+1} x^m P_n(x)dx = 2^{n+1}\frac{n!}{(2n+1)!}$, for $m = n$.

Q.2. (a) Derive Euler-Lagrange's differential equation by a variational procedure. (7)

OR

(a) Describe in detail Brachistochrone (shortest time) problem. (7)

(b) On the basis of electromechanical analogy, obtain Lagrangian for parallel L-C-R electric circuit. (5)

OR

(b) Explain the method of Lagrange's undetermined multiplier. (5)

Q-3 (a) Write the radial wave equation for interior region in three dimensional square well potential. Hence obtain its admissible solution. (6)

OR

Write the eigen value equation for the Anisotropic oscillator. Hence obtain its eigen functions and eigen values. (6)

(b) Write the radial wave equation for non-localized states (E > 0) of the three dimensional square well potential. Hence obtain its admissible solutions. (7)

OR

(b) Write the radial wave equation for hydrogen atom. Hence solve it to obtain its eigen values. (7)

Q-4 (a) Define eigen values and eigen functions of dynamical variable $\hat{A}$ in Hilbert space. Using ket vectors prove that the eigen value of a self adjoint operator is real, and the eigen functions belonging to different eigen values of a self adjoint operator are orthogonal. (6)

OR

(a) Define self adjoint operator. Discuss about why the self the adjoint operator in Hilbert space is also called the Hermitian operator? Hence discuss the unitary operator in brief. (6)

(b) Explain the effect of unitary translations on a wave function after translation of coordinate system. (6)

OR

(b) Explain the effect of transformation of coordinate system on a dynamical variable. Obtain the new dynamical variables $\hat{p}_x'$ and $\hat{L}_z'$ after the rotation of coordinate system about z-axis. (6)

Q-5 Answer in brief: (10)

(i) Write Hamilton's principle.

(ii) Define phase space.

(iii) What is a 'Hodograph'?

(iv) Write an expression for projection operator, for continuous basis, in Hilbert space?

(v) For continuous bases in Hilbert space $<x|x>=$ ………

(vi) Write any two conditions that must be satisfied by the state vectors in Hilbert space.

(vii) The generating function for $J_n(x)$ is…………..

(viii) The generating function for $P_l(x)$ is…………..

(ix) The spherical Bessel and Neumann functions are ……. and ……...

(x) The eigen value of Isotropic oscillator is $E_n =$…………

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